MATH 2050C Mathematical Analysis I
2022-23 Term 2
Problem Set 6
due on Mar 3, 2023 (Friday) at 11:59PM

Instructions: You are allowed to discuss with your classmates or seek help from the TAs but you are required to write/type up your own solutions. You can either type up your assignment or scan a copy of your written assignment into ONE PDF file and submit through Gradescope on/before the due date. Please remember to write down your name and student ID. **No late homework will be accepted.** All the exercises below are taken from the textbook.

Required Readings: Chapter 3.3

Optional Readings: none

Problems to hand in

Section 3.3: Exercise # 4, 7, 10, 12(b)

Suggested Exercises

Section 3.3: Exercise # 1, 2, 3, 5, 6, 8, 9, 11, 12(a)(c)(d)

Challenging Exercises (optional)

1. Let $x_1 := \sqrt{2}$ and for $n \geq 2$,
$$x_n := \sqrt{2 + \sqrt{x_{n-1}}}.$$
Prove that (x_n) is convergent.

2. Let $a > \sqrt{2}$ be a fixed number. Define $x_1 := a$, and for $n \geq 2$,
$$x_n := \frac{2 + x_{n-1}}{1 + x_{n-1}}.$$

(a) Show that $x_1 > x_3 > x_5 > \cdots$.

(b) Show that $x_2 < x_4 < x_6 > \cdots$.

(c) Prove that $\lim(x_n) = \sqrt{2}$.